Estimation of directional wave spectrum using a wave-probe array.


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Abstract

This paper discusses the evaluation of a series of multidirectional waves generated at the LabOceano wave basin. The methodology adopted follows the one proposed by Stansberg (1998), using an array of eight wave-probes. A computational routine was developed for the estimation and a preliminary validation was obtained in a numerical analysis. In this case, the wave spectra were known, and problems related to measurement noise were not considered. Such preliminary analysis showed that the method was able to estimate the directional spectrum precisely, considering short and long crested unimodal waves and also bimodal waves. The statistical parameters (significant wave height, peak period), mean direction and spreading functions were estimated with very good accuracy. The method was then applied to a full set of experiments in the LabOceano wave tank. Short and long crested unimodal waves and also bimodal waves were generated. The maximum entropy inference method was then used to estimate the directional wave spectrum, and to compare it with the spectrum used as the input for the wave generator. The results showed a very close proximity between required and measured results, both for statistics and directional spreading functions.

Keywords

Directional wave spectrum ; experimental investigation; ave-prove array; Maximum Entropy Method

1 Introduction

Most of the criteria employed nowadays for the design of floating offshore units rely on the analysis of unidirectional (long-crested) seas, although real wave spectra usually present a significant level of directional spreading and quite often result of a combination of seas with different mean directions. In recent years, however, with the ongoing expansion of laboratorial facilities around the world capable of generating multidirectional waves, research on the effects of wave directionality is increasing.

The ocean tank in LabOceano is one of the most recent examples of such facilities, being able to model a large variety of multidirectional waves. Wave generation is based on an in-line array of 75 flaps which allow emulating short-crested seas or the combination of two different unidirectional seas. The input may be based on a theoretical spreading model (as cos2s, for example) or also by directly defining the energy value for different combinations of wave frequency and direction.

This paper evaluates the characteristics of several different multidirectional irregular waves generated in the tank, comparing the measured spectra with the predicted ones used as input for the generation. To perform such evaluation, it was necessary to define a reliable method for inferring the experimental sea spectra, which, by itself, is not an easy task. A lot of research has been done on the evaluation of multidirectional seas based on wave-probe measurements and several difficulties were faced and discussed. Brigs (1997), for example, summarizes the results of IAHR Working Group on Multidirectional Waves, a joint effort of different tanks to address this particular matter. Benoit et al. (1997) reviewed directional analysis methods for linear waves and concluded that the efficiency of the analysis depends on the measuring devices and on the properties of the directional wave spectra. Important questions concerning statistical variability and directional resolution were also studied by Stansberg (1998).

In the present work, a statistical inference method based on an array of wave probes was employed. The procedure adopted was the one originally suggested by Stansberg (1998), which makes use of an array composed of eight wave probes. Statistical inference is performed based on the Maximum Entropy Method (MEM), as presented, for example, in Ochi (1998).

The Numerical Offshore Tank group of the University of São Paulo (USP) was responsible for defining the methodology and providing the computational code employed in the analysis. The LabOceano staff worked on the required instrumentation and performed the wave calibration and generation. Tests were sponsored by Petrobras and all the calibrated waves were subsequently employed for testing an FPSO model as part of an ongoing research project conducted by USP and Petrobras for evaluating the feasibility of estimating the same wave spectra based on the FPSO unit motions.

Tests involved extreme seas with different levels of wave spreading, combined long-crested seas with distinct mean directions and also the emulation of real seas measured by a wave-buoy. All the spectrum parameters were based on real oceanographic data of Brazil's Campos Basin.

2 Directional spectrum estimation by maximum entropy method

Directional wave spectrum $S(\omega,\theta)$ of a multidirectional wave field may be defined as:

$$S(\omega,\theta) = S(\omega)D(\omega,\theta) \tag{1}$$

where $S(\omega)$ is the wave power spectrum and $D(\omega,\theta)$ is the spreading function. The spreading function is normalized by:

$$\int_0^{2\pi} D(\omega,\theta)d\theta = 1 \tag{2}$$

The problem addressed in the rest of the paper is the estimation of $D(\omega,\theta)$ from records of the time series of the wave elevation at an array of measuring points. In the present study, the Maximum Entropy Method (MEM) has been applied, as proposed in Stansberg (1987). In order to apply such statistical inference method, the spreading function is written, for each frequency, as:

$$D(\theta/\omega) = \exp\left\{-1+\sum_{j=1}^{M+1}\lambda_j q_j(\theta)\right\} \tag{3}$$

where M=N.(N-1) and N is the number of wave probes. The Lagrange Multiplies (λ_i) are unknown, and may be obtained by the solution of the following set of equations:

$$\frac{S_{mn}(\omega)}{S(\omega)} = \int_{-\pi}^{\pi} q_j(\theta)D(\theta/\omega) \quad \text{for j} = 1,2,...,\text{M}+1 \tag{4}$$

being $S_{mn}(\omega)$ the cross spectra of the wave elevation measured at probes m and n. The wave power spectrum $S(\omega)$ is estimated as the average of the power spectrum obtained from all wave probes signals. In equations (3) and (4), the functions $q_j(\theta)$ are given by:

$$q_j(\theta) = \begin{cases} \cos\{kr_j\cos(\beta_j-\theta)\} & \text{for j} = 1,2,...\text{M}/2 \\ \sin\{kr_j\cos(\beta_j-\theta)\} & \text{for j} = \text{M}/2+1,...,\text{M} \\ 1 \quad \text{for j} = M+1 \end{cases} \tag{5}$$

with:

$$\begin{aligned} r_j &= \{(x_n-x_m)^2+(y_n-y_m)^2\}^{1/2} \\ \beta_j &= \tan^{-1}\{(y_n-y_m)/(x_n-x_m)\} \end{aligned} \tag{6}$$

where (x_i, y_i) is the position of the wave probe i. So, the Lagrange Multipliers λ_i are obtained by the solution of the algebraic equations (4). The spreading function is then obtained using equation (3).

The lay-out of the probes used in the present work was originally proposed by Stansberg (1998). An arrangement of 8 wave-probes is employed, being 7 of them equally distributed around a 0.5m radius circumference and the other one in the central point of the circumference (Figure 1).

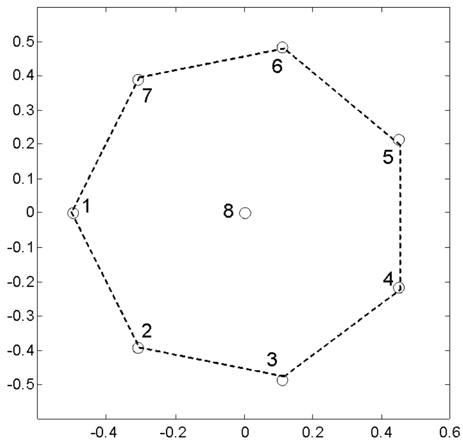

Fig. 1 Circular wave probes array

3 Applications

3.1 Numerical simulation

In order to evaluate the wave probe array lay-out and the estimation procedure defined in the previous section, a numerical analysis was carried out.

The time series of wave elevation in each wave probe was numerically calculated, considering the time realization of a particular directional wave spectrum. The wave elevation $\eta_i(t)$ measured at probe *i* is therefore given by:

$$\eta_i(t) = \int_{-\infty}^{+\infty}\int_{-\pi}^{\pi} \cos\{kx_i\cos\theta + ky_i\sin\theta - \omega t + \varepsilon\}\sqrt{2S(\omega,\theta)d\omega\,.d\theta} \quad (7)$$

where *k* is the wave number ($k = \omega^2/g$)

After the generation of time series $\eta_i(t)$ the MEM method described in the previous section was applied, and the obtained spectrum parameters were compared to the theoretical ones (those used to generate the time series $\eta_i(t)$ in equation (7)).

Several wave unimodal directional spectra were considered. All cases confirmed the efficiency of the MEM method, with a very close proximity between the estimated and theoretical parameters of the wave spectrum. For example, consider the typical sea state presented in Table 1, with parameters given both in real and model scale (1:70).

Table 1. Unimodal Sea State - Numerical Simulation

Wave Parameters	Real Scale	1:70 Model Scale
Significant Height (H_s)	4.5m	0.064m
Peak Period (T_p)	10.29s	1.23s
γ Factor	1.51	

Table 2 presents the results of the application of MEM method to the wave spectrum estimation, considering three differentcases. The "A" case corresponds to the sea state defined in Table 1 with a small spreading, what is represented by the large value of the parameter *s*. The "B" case considers the sea with a larger angular spreading (*s* = 12). The last case corresponds to a rotation of 45º in the mean wave direction. It can be verified that the wave height, period and direction are estimated with a very good accuracy. The spreading parameter (*s*) and form factor g present somewhat larger estimation errors, but this, in fact, could be anticipated since these particular parameters are very sensitive to small variations of the wave spectrum.

Table 2. Unimodal Sea State Estimation by MEM method - Numerical Simulation

	Spectrum used to generate the elevation time series					Spectrum obtained after MEM application				
Sea	Hs(m)	Tp(s)	γ	s	θ(degrees)	Hs(m)	Tp(s)	γ	s	θ(degrees)
A	0.064	1.23	1.51	60	0.00	0.067	1.25	1.05	80.00	0.13
B	0.064	1.23	1.51	12	0.00	0.063	1.25	0.98	27.84	0.32
B 45o	0.064	1.23	1.51	12	45.00	0.064	1.25	1.33	34.02	46.00

"B" case is detailed in the next figures. Figure 2 shows the power spectrum *S*(*ω*), and it can be verified the close proximity between the theoretical and estimated functions. Such small (and not important) difference between the estimated and theoretical functions *S*(*ω*) causes the large discrepancy in the g factor estimation (see Table 2).

Figure 3 shows the spreading function D(*ω*,*θ*) for several values of the frequency w. Despite of the fact that parameter *s* was not estimated with the same level of accuracy, the spreading function estimation is acceptable for frequencies higher than 4rad/s. It must be stressed that the spectrum presents small amount of energy for frequencies smaller than 4rad/s.

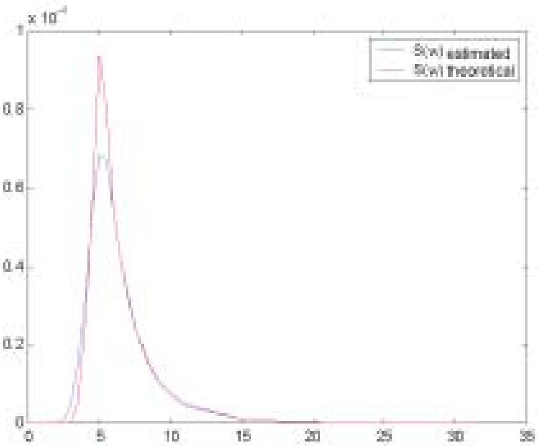

Fig. 2 Estimated and Theoretical power spectrum – numerical simulation

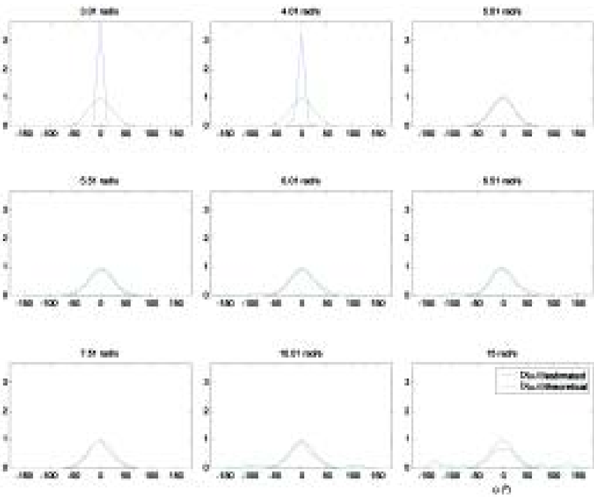

Fig. 3 Estimated and Theoretical spreading functions – numerical simulation

Several bimodal waves were also considered, for which the proposed method also presented good results. For example, let's consider the wave spectrum composed by the association of "wave 1" and "wave 2" presented in Table 3. The power spectrum and spreading functions comparisons are presented in Figures 4 and 5. Once again, a very good adherence is verified.

Table 3. Bimodal Sea State – Numerical Simulation

Wave 1				Wave 2			
Tp (s)	Hs (m)	θ	γ	Tp (s)	Hs (m)	θ	γ
1,38	0,021	0º	1,4	0,79	0,014	60º	0,9

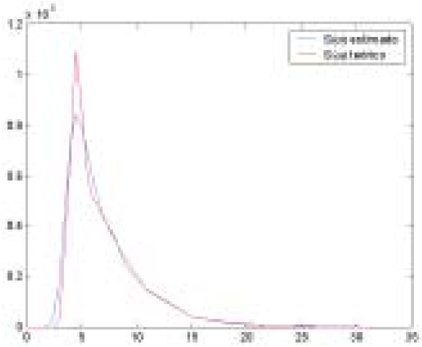

Fig. 4 Estimated and Theoretical power spectrum – bimodal wave – numerical simulation

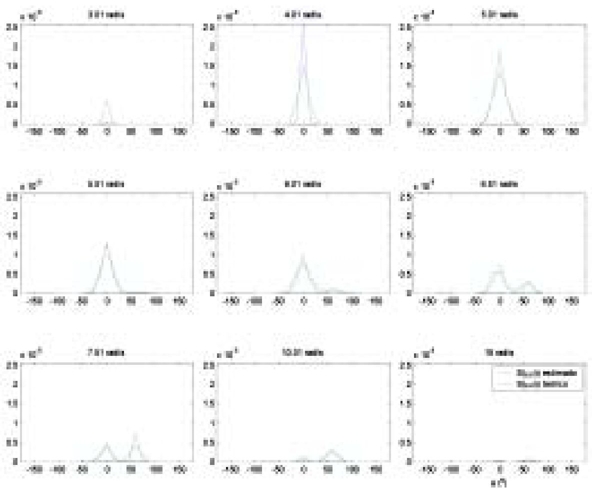

Fig. 5 Estimated and Theoretical spreading functions - bimodal wave – numerical simulation

3.2 Experimental analysis

The method was then applied to an entire set of experiments in the LabOceano wave tank. Short and long crested unimodal waves and also bimodal waves were generated.

In such tests, a directional wave spectrum is the input for the wavemakers control system that uses a proprietary algorithm to control the motion of each wave flap. The MEM estimation method was then used to estimate the directional wave spectrum and to compare it with the spectrum used as the input for the wave generation.

Power and cross spectra are evaluated from time series of wave elevation, with approximately 1350s (25Hz sample rate). A Fast Fourier algorithm is applied, with a length of 2048 and a Hanning Window Function with parameter n=1024, with overlap of 512 samples.

Unimodal waves

Table 4 presents the result of the application of the MEM method to the wave spectrum estimation, considering the entire set of unimodal wave spectra considered in the experimental analysis. Four groups of waves can be identified. The "Real Case Long Crested" group corresponds to two wave spectra measured in Campos Basin by means of a wave-buoy that were reproduced in LabOceano wave tank in the 1:70 model scale. For this first group, however, no directional spreading was applied to the waves, reproduction been based exclusively on the power spectra. The "Real Case Short Crested" corresponds to the real waves of the previous group, considering now the spreading as estimated by the wave-buoy. The third and fourth groups, named as "Irregular Waves", contain four different waves generated by means of the JONSWAP spectrum formulation in two different model scales, 1:70 and 1:48. For each group, two different peak periods were considered (10.3s and 15.3s in real scale). Long-crested (LC) and short-crested seas (s=60;12) were tested.

Table 4. Unimodal Sea State Estimation by MEM method – Experimental Analysis

		Theoretical					Experimental					Difference				
Name	Description	Hs(m)	Tp(s)	teta (o)	s	gama	Hs(m)	Tp(s)	teta (o)	s	gama	Hs	Tp	teta (o)	s	gama
W02_11201	Real Case Long Crested	0.024	1.09	0.00			0.026	1.08	0.22	85.0	1.94	11.0%	0.6%	0.2		
W07_10500		0.028	1.41	0.00			0.032	1.25	0.69	85.0	1.42	14.9%	10.8%	0.7		
W07_10600	Real Case Short Crested	0.024	1.09	-0.69			0.024	1.05	-1.74	32.3	2.81	3.0%	3.7%	1.1		
W07_10102		0.028	1.41	4.72			0.029	1.36	7.27	18.7	2.21	3.3%	3.5%	2.5		
W07_10202	Irregular Waves 1:70 Scale	0.064	1.23	0.00	LC	1.51	0.066	1.21	-0.14	85.0	1.73	2.5%	1.6%	0.1		14.6%
W07_10301		0.111	1.83	0.00	LC	1.70	0.115	1.74	1.08	36.2	1.86	3.6%	4.9%	1.1		9.4%
W07_10401		0.064	1.23	0.00	60	1.51	0.067	1.21	0.70	53.9	1.75	4.2%	1.6%	0.7	10.2%	15.9%
W07_20104		0.064	1.23	0.00	12	1.51	0.066	1.21	2.44	11.8	1.57	3.4%	1.6%	2.4	1.7%	4.0%
W07_20203	Irregular Waves 1:48 Scale	0.094	1.49	0.00	LC	1.51	0.090	1.57	0.94	46.2	1.69	3.8%	5.4%	0.9		11.9%
W07_20301		0.163	2.22	0.00	LC	1.70	0.168	2.24	2.17	48.5	1.37	2.8%	0.7%	2.2		19.4%
W07_20401		0.094	1.49	0.00	60	1.51	0.096	1.43	0.44	31.7	1.03	2.6%	4.4%	0.4	47.1%	31.8%
W02_12101		0.094	1.49	0.00	12	1.51	0.095	1.49	1.35	13.2	1.62	0.6%	0.0%	1.3	10.1%	7.3%

The same conclusions previously obtained for the numerical analysis can be derived from the experimental results (Table 4). The estimation errors for the wave height, period and direction are small. The spreading parameter s and γ factor present larger estimation errors[1]

Figure 6 and 7 illustrate the results obtained for the cases W07_10401 and W07_20401. They correspond to the same real scale sea state considered in the numerical analysis (Table 1) for reduced scales 1:70 and 1:48. The directional spreading in these cases is large (s=12). It can be seen that, despite a slightly better power spectrum estimation in the case of the 1:48 scale, both sea spectra are estimated with very good accuracy, confirming the applicability of the proposed wave array arrangement for the estimation of the short-crested seas generated in the LabOceano wave tank.

The accuracy in the statistical parameters is similar to the one verified for the long-crested situations. To illustrate this fact, Figure 8 presents the results for the test W07_10102, which correspond to the same power spectrum of Figure 6, now with a smaller directional spreading (s=85). Agreement between required and estimated spectra is again very good.

Figure 9 presents the case W07_10600, which correspond to one of the wave spectra obtained from wave-buoy measurements. In this case, input for wave generation corresponded to a matrix containing the values of the wave spectrum for different combinations of frequency and direction. It can be seen that a good reproduction of the power spectrum is obtained, with a slight distortion of the directional distribution. This distortion results in the 3,5o difference in mean direction.

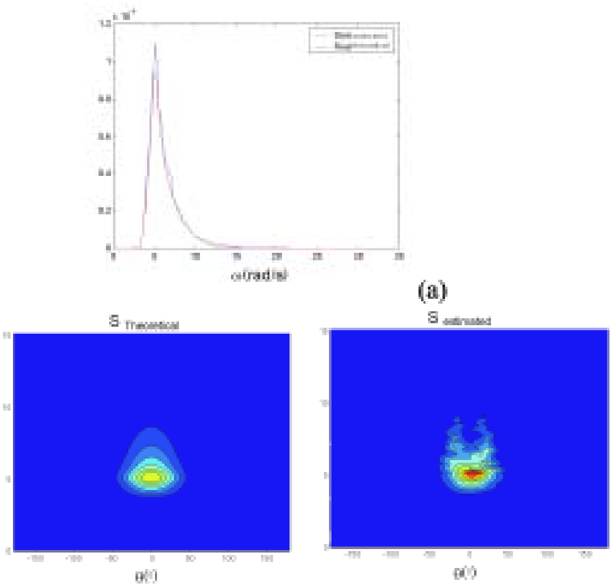

Fig. 6 Case W07_10401 – Scale 1:70 – (down) color maps of $S(\omega,\theta)$

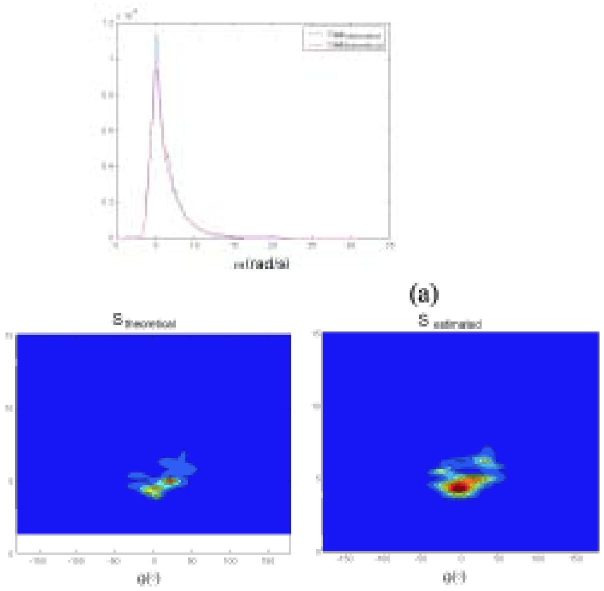

Fig. 8 Case W07_10102 – Small Spreading

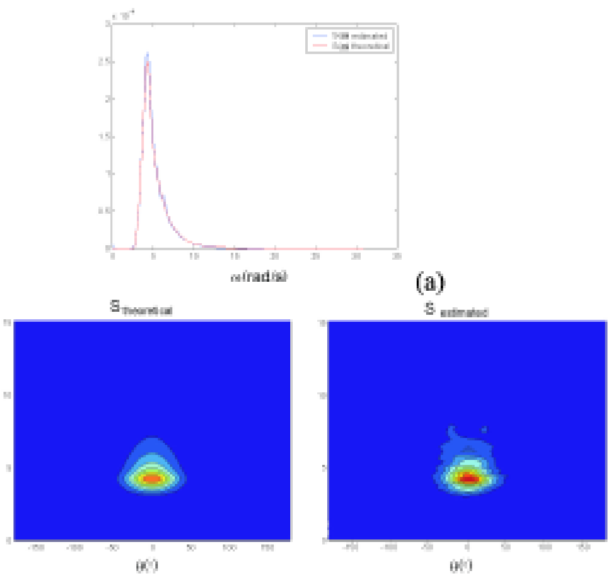

Fig. 7 Case W07_20401 – Scale 1:48

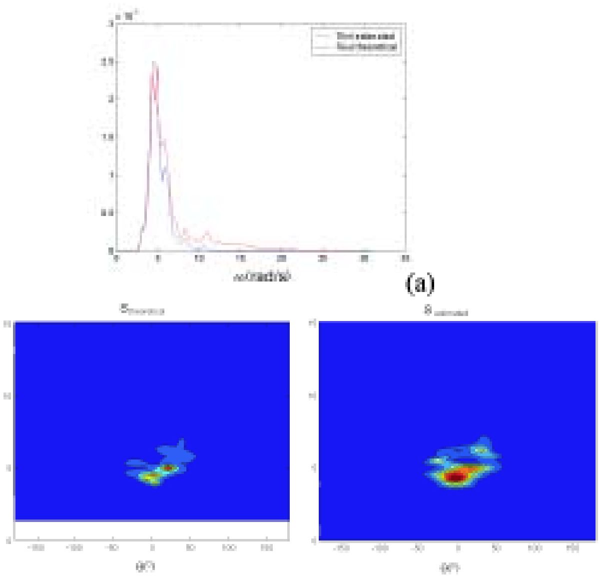

Fig. 9 CaseW07_10600 – Real Unimodal Case

1 Due to numerical approximations, the estimation of the spreading parameter s is limited by $s \leq 85$

Bimodal Waves

Several bimodal sea states were computed using a combination of two JONSWAP waves coming from different directions, with an angle up to 60o between mean directions. Such sea states were also generated for both, 1:70 and 1:48 model scales. Furthermore, a real bimodal sea spectrum, obtained from the wave-buoy, was reproduced in the wave tank in the 1:70 scale. The average statistics of each bimodal spectrum were calculated (Significant Height, Peak Period and Mean Direction), and the comparison between estimated and theoretical values are shown in Table 5.

Table 5. Bimodal Sea State Estimation by MEM method – Experimental Analysis

		Theoretical			Experimental			Difference		
bimodal	Description	Hs	Tp	teta	Hs	Tp	teta	Hs	Tp	teta
W02_10109	1:70 Scale	0.025	1.38	-12.86	0.021	1.36	-2.03	16.3%	1.2%	10.8
W07_10200		0.031	0.87	-23.69	0.026	0.87	-13.02	16.7%	0.2%	10.7
W07_10300		0.023	0.64	-18.83	0.014	1.36	0.44	39.0%	113.0%	19.3
W07_10400		0.032	1.37	-18.95	0.030	1.36	-1.31	6.2%	0.5%	17.6
W07_10500		0.025	1.38	-27.45	0.021	1.36	-14.95	15.5%	1.2%	12.5
W07_10601	Real Case	0.034	0.92	-1.26	0.022	0.92	1.75	33.3%	0.4%	3.0
W07_20100	1:48 Scale	0.037	1.67	-11.80	0.034	1.65	0.80	9.6%	1.2%	12.6
W07_20200		0.045	1.06	-24.34	0.040	1.08	-10.52	9.4%	1.9%	13.8
W07_20300		0.034	0.77	-20.18	0.025	0.77	0.95	24.4%	0.0%	21.1
W07_20400		0.047	1.65	-18.73	0.045	1.65	-0.66	5.1%	0.0%	18.1
W02_20500		0.037	1.67	-28.89	0.033	1.74	-14.38	11.5%	4.2%	14.5

The analysis of bimodal waves shows that the differences between the theoretical and estimated values are somewhat larger than for the unimodal cases. This can be to some extent explained by an imperfect absorption at the lateral beach of the tank, which may cause a larger distortion in this case, since there are important wave components in several directions. Also, according to the Biesel limit, the generation of high frequency components is now impaired, especially when scale reduction is more pronounced (waves were generated for +/- 30o angles). Comparison of the results of W06_10300 and W06_20300 (Figures 10 and 11) illustrate the discussion. Such cases correspond to the same sea state, in the 1:70 and 1:48 scale factors. There is a large and fast wave (H_s=1.47m; T_p=5.35s in real scale) combined to a smaller and slower wave (H_s=0.63m; T_p=11.30s), with a 60o angle between mean directions. Observing the power spectra of both cases, it is evident that lower peak period component is beyond the limits of wave generation, especially for the 1:70 scale case (Figure 10).

Test W06_10109, on the other hand, presents a quite different situation, and is presented in Figure 12. The wave field combines a higher and slower sea (H_s=1.47m; T_p=11.54s) to a smaller and faster one (H_s=0.98m; T_p=6.6s), once again with an angle of 60o between mean directions. In such case, wave energy is concentrated in a lower frequency range, and therefore both components can be generated with better accuracy.

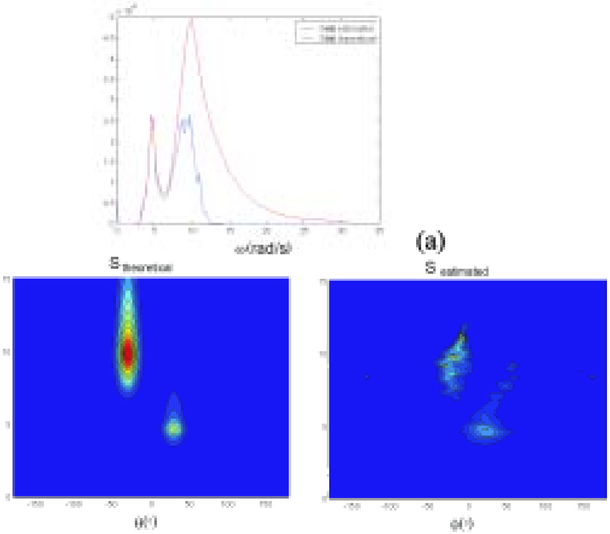

Fig. 10 Case W06_10300 – Scale 1:70

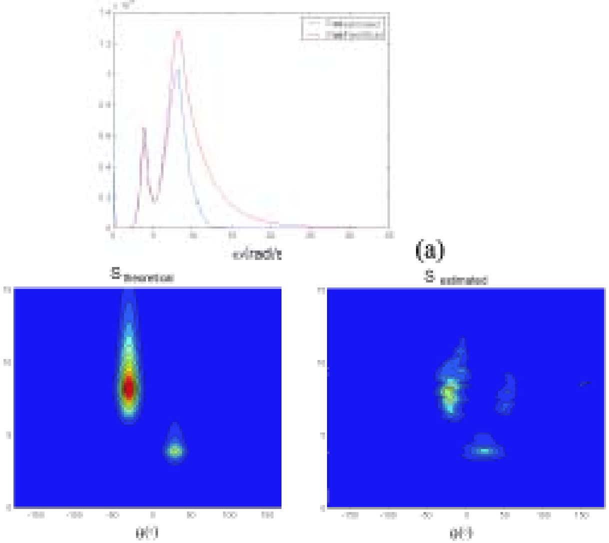

Fig. 11 Case W06_20300 – Scale 1:48

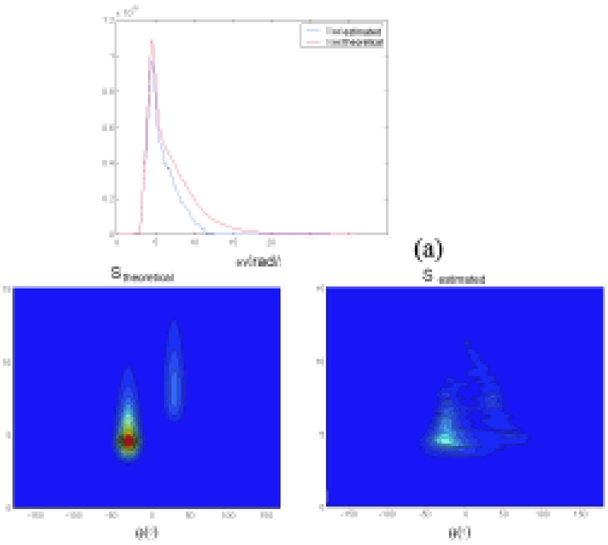

Fig. 12 Case W06_10109

Finally, Figure 13 presents the case W06_10601, a real bimodal sea state measured in Campos Basin by mean of a wave-buoy. Even though high frequency waves are once again beyond the limits of generation, resulting in a somewhat lower significant height in the tank, mean directions of both peaks are correctly reproduced.

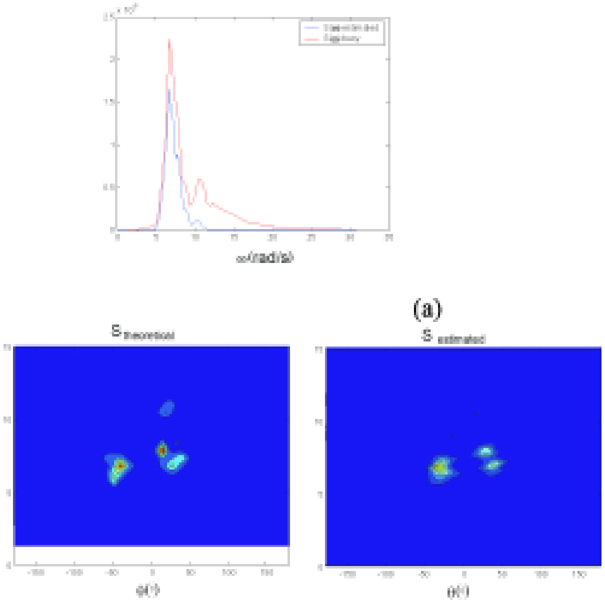

Fig. 13 Case W06_10601 – Real Bimodal Case

4 Conclusions

A series of multidirectional waves generated in the LabOceano wave basin was analyzed. Tests comprised long and short-crested seas and also the direct reproduction of wave spectra measured by a wave-buoy. The procedure employed for estimating the wave fields in the tank followed the method proposed by Stansberg (1998), using an array of eight wave probes. Statistical inference is based on the MEM method.

Overall results showed that most cases could be accurately generated in the wave tank, with a good reproduction of wave height, period and directional spreading. Large discrepancies were verified only for some bimodal spectra with significant energy beyond the limits of the generation system. Apart from these situations, results proved that a wide range of multidirectional irregular waves could be well reproduced in the basin.

The good agreement between measured and specified results also attested that the estimation method employed represents an efficient procedure for future analysis of multidirectional wave tests in the LabOceano basin, provided that the properties of the wave fields remain within the range of the conditions already tested.

5 Acknowledgments

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